

Stateman: Using Metafunctions to Manage Large Terms Representing Machine States

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When ACL2 is used to model the operational semantics of computing machines, machine states are typically represented by terms recording the contents of the state components. When models are realistic and are stepped through thousands of machine cycles, these terms can grow quite large and the cost of simplifying them on each step grows. In this paper we describe an ACL2 book that uses HIDE and metafunctions to facilitate the management of large terms representing such states. Because the metafunctions for each state component updater are solely responsible for creating state expressions (i.e., “writing”) and the metafunctions for each state component accessor are solely responsible for extracting values (i.e., “reading”) from such state expressions, they can maintain their own normal form, use HIDE to prevent other parts of ACL2 from inspecting them, and use honsing to uniquely represent state expressions. The last feature makes it possible to memoize the metafunctions, which can improve proof performance in some machine models. This paper describes a general-purpose ACL2 book modeling a byte-addressed memory supporting “mixed” reads and writes. By “mixed” we mean that reads need not correspond (in address or number of bytes) with writes. Verified metafunctions simplify such “read-over-write” expressions while hiding the potentially large state expression. A key utility is a function that determines an upper bound on the value of a symbolic arithmetic expression, which plays a role in resolving writes to addresses given by symbolic expressions. We also report on a preliminary experiment with the book, which involves the production of states containing several million function calls.

1 Background

ACL2 [3, 2] is frequently used to model computing machines via operational semantics. It is not difficult to configure the ACL2 theorem prover so that it can use the definitions of the machine semantics and a few well-chosen rewrite rules to step through code sequences, split on tests, induct on loops, etc. Examples of these methods being used to prove functional correctness of code under formal operational semantics may be found in numerous publications [6, 7, 10, 1]. Such symbolic state terms can grow quite large when many steps are composed. The question addressed here is: *how can we exploit ACL2’s rewriter to symbolically execute formalized code while preventing it from slowing down as state expressions get large?*

This paper describes the Stateman book for managing large terms representing machine states in ACL2 models of computing machines. “Stateman” stands for “state management.” This is a work in progress and this paper has many brief descriptions of intended **Future Work**.

The idealistic dream is that a user wishing to model some byte-addressed computing machine and do code proofs or run the Codewalker tool¹ might build the operational semantics on top of the state

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¹Codewalker extracts ACL2 functions from machine code given the formal operational semantics of the ISA and is sim-

provided by Stateman and thereby inherit the state management techniques here described. But machine models are very idiosyncratic. Users may actually need to design their own states and merely exploit the basic techniques described here. Thus, this paper focuses mainly on the design decisions in our work. As usual, readers are welcome, indeed encouraged, to read the Stateman book itself and use it as the basis of their own versions.

We start with a brief description of our generic state, then we present the highlights of our state management techniques, provide some examples, discuss a few details, and present some preliminary performance measures.

2 The Generic State

The book provides a generic single-threaded object, ST (henceforth, *st*), providing three fields. See :DOC stobj.²

```
(defstobj st
  (I :type unsigned-byte :initially 0)          ; program counter
  (S :initially nil)                             ; status
  (M :type (array (unsigned-byte 8) (*m-size*)) ; memory
    :initially 0
    :resizable nil
  )
  :inline t
  :renaming
  ((UPDATE-I   !I)
   (UPDATE-S   !S)
   (UPDATE-MI  !MI)
   (M-LENGTH   ML)))
```

The primitive accessors are I, S, and MI, and the primitive updaters are !I, !S, and !MI.³ The I and S fields were originally intended for the machine’s instruction counter and status flag, and MI provides a byte addressed memory of 8-bit bytes. The person using this book to model the state of a computing machine need not use the I and S fields for their implied purposes. The modeler might, for instance, choose to store all state information including the instruction counter and various status bits in the byte addressed memory and ignore the I and S fields altogether.

Byte-addresses are integers starting at 0. The byte-addressed memory is of fixed size, **m-size**, which is currently only 5312. This constant is a holdover from the earliest use of the state and (**Future Work**) will be generalized in future work. Indeed, the whole development would have been easier were there no upper bound on memory size. Imposing an upper bound forced certain issues to be dealt with –

ilar to the HOL decompilation work by Magnus Myreen[8, 9]. See the README file in the Community Book directory `projects/codewalker/`. The version of Codewalker used here is still experimental.

²When we say “See :DOC *x*” we mean see the documentation topic *x* in the ACL2 documentation, which may be found by visiting the ACL2 home page[4], clicking on [The User’s Manuals](#), then clicking on the [ACL2+Books Manual](#) and typing *x* into the “Jump to” box.

³The third field of the single-threaded object is named M and is an array, but only the elements can be accessed or changed, with MI and !MI.

issues that are necessarily raised in any realistic model. The magnitude of that upper bound is practically irrelevant from the research perspective.

The Stateman book uses MI and !MI only to provide support for two more general utilities, R and !R, for reading and writing an arbitrary number of bytes. We do not think of MI and !MI as “visible” to the user of Stateman.

It is best to think of the generic state as providing the following functionality:

expression	value
(I <i>st</i>)	instruction counter of state <i>st</i>
(S <i>st</i>)	status flag of state <i>st</i>
(R <i>a n st</i>)	natural number obtained by reading <i>n</i> bytes starting at address <i>a</i> in the memory of state <i>st</i>
(!I <i>v st</i>)	new state obtained from state <i>st</i> by setting the instruction counter to <i>v</i>
(!S <i>v st</i>)	new state obtained from state <i>st</i> by setting the status flag to <i>v</i>
(!R <i>a n v st</i>)	new state obtained by writing <i>n</i> bytes of natural number <i>v</i> into the memory of <i>st</i> starting at address <i>a</i>

R and !R use the “Little Endian” convention. For example, (!R *a n v st*) writes the less significant bytes of *v* to the lower addresses, with the least significant byte written to address *a* and all other bytes written to larger addresses. (**Future Work**) We would like to support either Little or Big Endian conventions.

Nests of !I, !S, and !R applications are called *state expressions* or *state terms* because they denote machine states. Any term whose top function symbol is I, S, or R applied to a state expression is called a *read-over-write* expression. Any term whose top function symbol is !I, !S, or !R applied to a state expression is called a *write-over-write* expression. Of course, write-over-write expressions are themselves state expressions.

Our concern here is simplifying read-over-write and write-over-write expressions in support of code proofs and code walks. These issues are straightforwardly managed with rewrite rules. For example, the read over write expression (R 24 8 (!R 40 8 *v st*)) can be simplified to (R 24 8 *st*). But as state expressions grow large – and they can grow very large when long code sequences are involved – two problems crop up.

First, the rewriter tends to re-simplify parts of states that have already been simplified. Second, the traditional rewrite rules for handling byte-addressed memory involve backchaining to establish that byte sequences do not overlap. For example, the rewrite rules that replace (R *a n* (!R *b k v st*)) by (R *a n st*) have the hypotheses (natp *a*), (natp *b*), (natp *n*), (natp *k*), and either (< (+ *a n*) *b*) or (< (+ *b k*) *a*). The inequalities can get very expensive when *a* and *b* are large arithmetic expressions. Furthermore, *a* and *b* typically become large arithmetic expressions when the code being explored is doing indexed addressing (as in array access) and long code sequences are involved in the computation of the indices. Every read-over-write and write-over-write expression raises such an *overlap* question. Furthermore, a read of a deeply nested state expression typically raises an overlap question for each write in the nest. For speed we must answer overlap questions without resorting to heavy-duty arithmetic.

3 Highlights of Key Design Decisions

Some of the key decisions in the design of Stateman are listed and briefly elaborated below. In the next section, where we give examples, we discuss the implications of some of these decisions.

- Manage read-over-write and write-over-write expressions exclusively with metafunctions:** Stateman defines a metafunction for each of I, S, R, !I, !S, and !R. These metafunctions are named `meta-I`, `meta-S`, etc. Like all metafunctions, they take terms as input and yield possibly different terms as output.⁴ The metafunctions for R and !R are extended metafunctions and thus additionally take the metafunction context and ACL2 state as arguments. These two metafunctions only use the type-alist in the metafunction context and they ignore the ACL2 state. However, the biggest problem faced by these functions is the read-over-write overlap questions: “is one address less than another?”, given only the syntactic expressions representing the two addresses. This motivates the next item.
- Implement a syntactic interval inference mechanism:** Imagine a function that when given an arithmetic/logical term, can infer an upper bound. This is quite different functionality than normally found in ACL2. ACL2 can be configured to answer questions like “Is α less than 16?” but here we want a utility for answering “What number is α less than?” This functionality is especially important in codewalking unknown code. Suppose the code in question uses α as an index into some array at location *base*. What part of the state is changed if the code writes to $base + \alpha$? If you know enough about the code to know the bound on the array, you could undertake to prove that α is in bounds and thus conclude that only the array is affected by the write. But if you do not know much about the code, you need an inference mechanism to deduce a bound on α . Stateman provides a verified interval inference mechanism named `Ainni` which is discussed in more detail in Section 5.
- Implement syntactic means of deciding some inequalities:** Given `Ainni`, it is possible to implement the extended metafunction `meta-<` that takes an inequality and the metafunction context and decides many inequalities, $(\alpha < \beta)$, by computing intervals for α and β and comparing their endpoints, e.g., if the upper bound of α is below the lower bound of β , then the inequality is true. This can save backchaining into linear arithmetic on large arithmetic/logical expressions.
- Implement syntactic means of simplifying some MOD expressions:** In machine arithmetic, expressions of the form $(\text{MOD } \alpha \text{ ' } n)$ frequently arise, where n is some natural number. Some expressions of this sort can be simplified by syntactic means given the ability to infer bounds on α . See Section 6.
- Use syntactic means to decide overlap questions:** Suppose the type-alist tells us that the 32-bit word at address 8, i.e., $(\text{R } 8 \text{ 4 } st)$ is less than 16. Then a quick syntactic scan of the address expression $(+ \text{ 3200 } (* \text{ 8 } (\text{R } 8 \text{ 4 } st)))$ reveals that the value lies in the interval [3200, 3320] and so reading, say, 3 bytes from that address might touch any address in the interval [3200, 3322].
- Insist that all byte counts be quoted constants:** This facilitates the interval analysis mentioned above. We do not regard it as a restriction given Stateman’s intended application for code analysis. In most ISAs the number of bytes to be manipulated by an instruction is explicitly given in the instruction or else is fixed by the instruction or the architecture.

⁴Metafunctions traffic in fully translated terms but the examples in this paper generally show untranslated terms for readability.

- **Do not put nested !R-expressions into address order:** We leave the most recent writes at the top of the state expression under the assumption that program code tends to read from addresses recently written.
- **Eliminate perfectly shadowed writes:** When !R, with address a and byte count n , is applied to a state expression already containing an application of !R with address a with byte count n , Stateman eliminates the inner (earlier) one. Similar considerations apply to nested !I and !S calls. This reduces the size of the final state expression. But Stateman does not try to eliminate partially shadowed writes. We explain below.
- **Use hons rather than cons to create state expressions:** This means that if the same state expression is created along different paths of a code proof or walk, no additional space is allocated; furthermore, hons facilitates the use of memoization.
- **HIDE the state expressions produced by the metafunctions:** This ensures that no rewrite rule touches them. For example, if a machine model mentions an expression like

```
(!R 32 4 v
  (!R 8 4 (+ (R 8 4 st) 4)
    (!I 123
      (!S NIL st))))
```

as would happen if it set the status flag to NIL, the instruction pointer to 123, incremented the word at address 8 by 4 and wrote v to the word at address 32, then the inside-out rewriting of ACL2 would invoke the metafunctions for !S, then !I, and then !R (twice) and ripple a HIDE out so the final term would be as exactly as above but with a single HIDE around it at the top level. It would never be further simplified except by these metafunctions.

- **HIDE some values extracted by reads from hidden states to avoid re-simplifying them:** This is a controversial decision and is still quite unsettled. (**Future Work**) The issue is that over long codewalks (involving thousands of instructions) the expressions built up as values in the memory can be huge. By embedding extracted values in HIDE expressions, they are not re-simplified. The downside is that it can be impossible to decide simple tests because one does not know much about the hidden expressions. A compromise would be to bury the HIDEs several levels down in the extracted expressions, leaving the top few function symbols available. At the moment, all extracted values are hidden except constants and calls of R. This means that the metafunctions here must remove some HIDEs from values before storing them into memory.
- **Prove guards and well-formedness guarantees of the metafunctions:** ACL2 users should be well aware of the efficiency advantages of verifying the guards on functions used in heavy-duty computations. A less familiar topic, though, is discussed in the new feature documented in :DOC well-formedness-guarantee. It has long been the case that when a metafunction is applied the theorem prover checks that the result is a well-formed term, by running the function `term` on the output and the current ACL2 world. This hidden cost of metafunctions goes all the way back to the origin of ACL2 in 1989. However, when the output of a metafunction is huge, the well-formedness check can be expensive, and the basic supposition in the Stateman work is that state expressions are huge. A new feature of ACL2 Version 7.2 makes it possible to skip the well-formedness check by *proving* that the metafunction always returns a `term`. We have found that providing such well-formedness guarantees is worthwhile in Stateman. See [5]. We give some data on this below.

4 Examples

We illustrate these ideas with a few examples. The reader may notice two odd aspects to our examples. The first is that most addresses illustrated are quoted constants. The second is that when non-constant expressions occur as addresses the only variable involved is *st* and it always occurs in a primitive state accessor like (R *a n st*). We do not believe these are serious constraints if Stateman is used for code analysis: Typical code, especially binary machine code, refers to fixed addresses or offsets from other addresses (as in array indexing and stack slots relative to some stack or frame pointer in a register); “variables” are just the contents of memory locations at such addresses. However (**Future Work**) it would not be difficult to support variable symbols provided the context established natural number bounds on their values.

Examples (1)–(7) below are extracted verbatim from a session log that started in a fresh ACL2 with the inclusion of the Stateman book. Because this is a work in progress, we keep the version number as part of the book name right now. This log started by including `stateman22.lisp` which is included in the supplemental material. The supplemental material also includes `simple-examples.lisp`, a file (not a book) showing the actual input forms for these and some other examples in this paper. We hope those forms can help the user who wishes to extend Stateman’s functionality.

```
ACL2 !>(meta-!I '(!I '123 st)) ;(1)
(HIDE (!I '123 ST))
```

```
ACL2 !>(meta-!R '(!R '0 '4 (R '16 '4 st) (HIDE (!I '123 ST))) ;(2)
      nil state)
(HIDE (!R '0 '4 (R '16 '4 ST) (!I '123 ST))) ;(st')
```

```
ACL2 !>(meta-I ' (I (HIDE (!R '0 '4 (R '16 '4 st) (!I '123 ST)))) ;(3)
'123
```

```
ACL2 !>(meta-R ' (R '0 '4 (HIDE (!R '0 '4 (R '16 '4 st) (!I '123 ST)))) ;(4)
      nil state)
(R '16 '4 ST)
```

```
ACL2 !>(meta-R ' (R '2 '2 (HIDE (!R '0 '4 (R '16 '4 st) (!I '123 ST)))) ;(5)
      nil state)
(HIDE (ASH (R '16 '4 ST) '-16))
```

```
ACL2 !>(meta-R ' (R '8 '4 (HIDE (!R '0 '4 (R '16 '4 st) (!I '123 ST)))) ;(6)
      nil state)
(R '8 '4 ST)
```

```
ACL2 !>(meta-R ' (R '2 '4 (HIDE (!R '0 '4 (R '16 '4 st) (!I '123 ST)))) ;(7)
      nil state)
(HIDE (BINARY-- (ASH (R '4 '2 ST) '16)
      (ASH (R '16 '4 ST) '-16)))
```

In example (1) we call the metafunction for !I on the term (!I '123 st), just as the rewriter does when it encounters a !I-term. The result is a hidden state. Notice that metafunctions traffic in fully translated terms.

In example (2) we call the metafunction for !R on the !R-term that writes the 4-byte value of (R '16 '4 ST) to location 0 in the previously produced (now hidden) state. Note that the metafunction for !R takes two additional arguments, the metafunction context, in this case nil, and the ACL2 state object, since meta-!R is an extended metafunction. Again, nothing significant happens except the new state is hidden. Henceforth in this narrative we will refer to the state produced by (2) as st' .

In example (3) we use the metafunction for I to extract the instruction counter of st' .

In example (4) we use the metafunction for R to read (4 bytes of) the contents of address 0 in st' . The result is exactly what was written in (2) because it was 4 bytes long.

In example (5) we read the last two bytes of that previously written quantity, that is, we read 2 bytes starting at address 2 in st' . Two things are noteworthy. One is that it is reported as the 4-byte quantity that was written in (2), shifted down by 16 bits. The second is that it is hidden – the “controversial” decision.

In example (6) we read from an address above any affected by the write in st' . The result is whatever was there in the original state st .

In example (7) we read 4 bytes starting at address 2 in st' . This is a “mixed” read in the sense that the result involves the last two bytes from what was written at address 0 and the bytes that were at locations 4 and 5 of the original state st . It is expressed as a sum, with the latter bytes shifted up. Again, it is (controversially) hidden.

It is important to realize that all of these transformations are carried out by verified metafunctions without involving rewrite rules, linear arithmetic, or other heavy-duty theorem proving. Consequently, these transformations are very fast.

Since the I and S slots are unaffected by writes to memory and do not involve addresses or overlap issues our examples below focus on R- and !R-terms.

Henceforth, we will display untranslated terms for both input and output and will not exhibit the calls of the relevant metafunction. Instead, the reader should understand that the notation “ $\alpha \Rightarrow \beta$ ” means that α is transformed to β by the metafunction appropriate for the top function symbol of α . Since both meta-R and meta-!R take a metafunction context we make clear in the surrounding narrative what the context is. This only involves describing the governing assumptions (as encoded in the type-alist). Finally, instead of writing something like “ $\alpha \Rightarrow (IF \text{ hyp } \beta \ \alpha)$ ” we will generally write “ $\alpha \Rightarrow^\dagger \beta$ ” and describe the side condition *hyp* generated by the metafunction in the accompanying narrative. Recall that before such an α is replaced by β the rewriter must establish *hyp*.

Given a metafunction context in which the type-alist is empty, we can thus recap lines (1)–(7) above with:

(!I 123 st) ;(1)
 \Rightarrow
 (HIDE (!I 123 st))

(!R 0 4 (R 16 4 st) (HIDE (!I 123 st))) ;(2)
 \Rightarrow
 (HIDE (!R 0 4 (R 16 4 st) (!I 123 st)))

(I (HIDE (!R 0 4 (R 16 4 st) (!I 123 st)))) ;(3)
 \Rightarrow

123

```
(R 0 4 (HIDE (!R 0 4 (R 16 4 st) (!I 123 st)))) ;(4)
```

 \Rightarrow

```
(R 16 4 st)
```

```
(R 2 2 (HIDE (!R 0 4 (R 16 4 st) (!I 123 st)))) ;(5)
```

 \Rightarrow

```
(HIDE (ASH (R 16 4 st) -16))
```

```
(R 8 4 (HIDE (!R 0 4 (R 16 4 st) (!I 123 st)))) ;(6)
```

 \Rightarrow

```
(R 8 4 st)
```

```
(R 2 4 (HIDE (!R 0 4 (R 16 4 st) (!I 123 st)))) ;(7)
```

 \Rightarrow

```
(HIDE (+ (ASH (R 4 2 st) 16)
          (ASH (R 16 4 st) -16)))
```

Relatively little work is done on simplifying writes, aside from looking for shadowed writes to be deleted. For example, one might wonder at the simple

```
(!R 8 4 v st) ;(8)
```

 \Rightarrow

```
(HIDE (!R 8 4 v st))
```

since v might be too big to fit in 4 bytes. But instead of truncating v on write we do so on read:

```
(R 8 4 (HIDE (!R 8 4 v st))) ;(9)
```

 \Rightarrow

```
(HIDE (MOD (IFIX v) 4294967296))
```

Now let the metafunction context encode the assumption that $(R\ 16\ 4\ st)$ is less than 16. In the example below, we treat $(R\ 16\ 4\ st)$ as an index into a QuadWord array (8-byte per entry) based at address 3200.

```
(R (+ 3200 (* 8 (R 16 4 st))) 8 ;(10)
```

```
(HIDE (!R 3600 4 v (!R 8 4 w st))))
```

 \Rightarrow^{\dagger}

```
(R (+ 3200 (* 8 (R 16 4 st))) 8 st)
```

```
(!R (+ 3200 (* 8 (R 16 4 st))) 8 u ;(11)
```

```
(HIDE (!R 3600 4 v
```

```
(!R 8 4 w
```

```
(!R (+ 3200 (* 8 (R 16 4 st))) 8 x
st))))
```


\Rightarrow

```
(HIDE (!R (+ 3200 (* 8 (R 16 4 st))) 8 u
      (!R 3600 4 v
        (!R 8 4 w st))))
```

The “†” on the transformation in (10) indicates that a side condition was generated. That side condition is $(\leq (R\ 16\ 4\ st)\ 15)$, and it must be established before the replacement is made. Establishing such side conditions should be trivial since they are extracted from the type-alist in the metafunction context. Given that condition, we see that the 8-byte read at $(+ 3200 (* 8 (R\ 16\ 4\ st)))$ may only touch bytes in the interval $[3200, 3327]$. We discuss this interval analysis further below. But because of it, the metafunction can determine that neither of the two writes in the hidden state of (10) is relevant since the 4 bytes starting at 3600 are above the target interval and 4 bytes starting at 8 are below it.

Interestingly, no side condition is necessary on transformation (11). If $(R\ 16\ 4\ st)$ is sufficiently large the new write at $(+ 3200 (* 8 (R\ 16\ 4\ st)))$ *might* shadow out the write at 3600, but that does not matter because the new write is added at the top of the expression (chronologically after the write at 3600), so the answer above is correct. And, regardless of the magnitude of $(R\ 16\ 4\ st)$, the new write shadows out the earlier one at the exact same address and the earlier write can be dropped.

Our final example is contrived to show a mixed read that spans several chronologically separated writes. The empty metafunction context is sufficient for this example. We will ultimately read 8 bytes starting at address 3. But consider the writes that create the relevant memory. The write of 4 bytes of v at address 2 is partially shadowed by the write of 4 bytes of u at address 0. The writes at 14 and 19 are irrelevant because we only need bytes 3 through 10. The first byte of our answer is the high order byte of u written at address 3. The next two bytes are the two high order bytes of v at addresses 4 and 5. Then we get 3 bytes from the original st at addresses 6, 7, and 8, and finally we get the two low order bytes from w at addresses 9 and 10. We then assemble these 8 bytes using the Little Endian notation and put the final sum into ACL2’s term order.

```
(R 3 8 ;(12)
  (HIDE
    (!R 14 5 x
      (!R 0 4 u
        (!R 19 8 y
          (!R 9 2 w
            (!R 2 4 v st))))))))
```

\Rightarrow

```
(HIDE (+ (ASH (R 6 3 st) 24)
  (+ (MOD (ASH (IFIX u) -24) 256)
    (+ (ASH (MOD (IFIX w) 65536) 48)
      (ASH (MOD (ASH (IFIX v) -16) 65536) 8)))))
```

(Future Work) We are dissatisfied with the normal form of expressions denoting the results of mixed reads. To be more precise, we do not have enough experience with it yet to know whether it is sufficient for our purposes. The current implementation uses IFIX to convert terms to integer form as required by basic rules for ASH (if syntactic analysis cannot establish that the term returns an integer), uses MOD to truncate unneeded higher order bits, and uses ASH to shift bits into the right locations. The question however is this: Suppose such an expression is written to a memory location and then one must read a few bytes from it. The current metafunctions produce ASH/MOD-terms that could be further simplified.

But given the controversial decision to HIDE the complicated results of reads, that simplification should be done inside `meta-R`.

Stateman does not produce normalized states for at least two reasons. First, it does not put writes into address order. Second it does not eliminate partial shadows. Why bother to eliminate partially shadowed material if one can read out the answers if and when needed? This consideration is especially relevant since resolving a partial shadow generally makes the state syntactically *larger*, e.g., to resolve the shadowing of the write at 2 above one would replace $(!R\ 2\ 4\ v\ st)$ by the larger term $(!R\ 4\ 2\ (ASH\ (IFIX\ v\ -16))\ st)$. It is not clear this is an improvement. Furthermore, we suspect partial shadowing is fairly rare compared to “perfect shadowing” where the n bytes starting at address a are repeatedly reused for different n byte values.

(Future Work) But the lack of normalization raises the question of determining state equality. Stateman does not support state equality at the moment. But the plan is to support it by a metafunction that announces the equality of two states formed by different sequences of writes to the same initial state by checking that every read of every byte written to either state produces the same expression.

5 Ainni: Abstract Interpreter for Natural Number Intervals

Perhaps the most important idea to come out of this work so far is the development and verification of an ACL2 function that takes the quotation of a term together with a type-alist and attempts to determine a closed natural number interval containing the value of the term. This function is called *Ainni*, which stands for *Abstract Interpreter for Natural Number Intervals*. *Ainni* can be thought of as a “type-inference” mechanism for a class of ACL2 arithmetic expressions, except the “types” it deals with are intervals over the naturals.

Ainni explores terms composed of constants, the state st , and the function symbols $+$, $-$, $*$, R , `HIDE`, `MOD`, `ASH`, `LOGAND`, `LOGIOR`, and `LOGXOR`.⁵ **(Future Work)** This set of function symbols was determined by seeing what functions were introduced by the codewalk of a particularly large and challenging test program: an implementation of DES. Essentially, *Ainni* should support all of the basic functions used in the semantics of the ALU operations of the machine being formalized. We therefore anticipate that the list here will have to grow.

Ainni recursively descends through the term “evaluating” the arguments of function calls – only in this case that means computing intervals for them – and then applying bounders (see the discussion of “bounders” in `:DOC tau-system`) corresponding to the function symbols to obtain an interval containing all possible values of the function call. At the bottom, which in this case are calls of R , *Ainni* uses the type-alist to try to find bounds on reads that are tighter than the syntactically apparent $0 \leq (R\ a\ n\ st) \leq 2^{8n} - 1$. **(Future Work)** It is here, at the “bottom” of the recursion, that we could add support for variable symbols or unknown function symbols.

For example, consider the quotation of the term

$(+ 288 (* 8 (LOGAND 31 (ASH (R 4520 8 st) -3))))$.

In the absence of any contextual information, *Ainni* returns the natural number interval $[288, 536]$. The reasoning is straightforward: we know that $(R\ 4520\ 8\ st)$ is a natural in the interval $[0, 2^{64} - 1]$. The tau-bounder for `ASH` tells us that shifting it right 3 reduces that to $[0, 2^{61} - 1]$, and then the tau-bounder for `LOGAND` tells us that bitwise conjoining it with 31 shrinks the interval to $[0, 31]$. Multiplying by 8 makes the interval $[0, 248]$, and adding 288 makes it $[288, 536]$.

⁵Several of these symbols are macros that expand into calls of function symbols that *Ainni* actually recognizes.

By default $(R \ 4520 \ 8 \ st)$ is known to lie in $[0, 2^{64} - 1]$, but the type-alist might restrict it to a smaller interval. For example, it might assert that $(R \ 4520 \ 8 \ st) < 24$, in which case Ainni determines that the term above lies in the interval $[288, 304]$.

In addition to returning the interval, Ainni also returns a flag indicating whether the term was one that Ainni could confine to a bounded natural interval and a list of hypotheses that must be true for its interval to be correct. These hypotheses have two sources: (i) assumptions extracted from the context and (ii) Ainni's inherent assumptions (such as a built-in assumption that no computed value is negative⁶, which might translate to the hypothesis $(\text{not } (< \ x \ y))$ if the term is $(- \ x \ y)$).

Finally, Ainni is verified to be correct. That is, the certification of Stateman involves a proof of the formal version of:

Let x be the quotation of an ACL2 term and ta be a type-alist. Let flg , $(h_1 \ \dots \ h_k)$ and $[lo, hi]$ be the flag, hypotheses, and the interval returned by Ainni on x and ta . Then if flg is true:

- $(h_1 \ \dots \ h_k)$ is a list of quotations of terms,
- lo and hi are natural numbers such that $lo \leq hi$, and
- if $(\mathcal{E} \ h_i \ a) = T$ for each $1 \leq i \leq k$, then $lo \leq (\mathcal{E} \ x \ a) \leq hi$, where \mathcal{E} is an evaluator that recognizes the function symbols handled by Ainni.

Ainni is used in meta-R to handle the overlap questions that arise. In addition, it is used in meta-< to decide some inequalities and in meta-MOD to simplify some MOD expressions.

Furthermore, Ainni is fast. For example, in the codewalk of the DES algorithm, one particular index expression is a nest of 382 function calls containing every one of the function symbols known to Ainni. Just for fun, here is the expression, printed “almost flat” (without much prettyprinting):

```
(LOGIOR
 (LOGAND 32 (ASH (MOD (ASH (LOGXOR (LOGIOR (ASH (ASH (LOGIOR (ASH (MOD (ASH (R 4520 8 ST) 0) 2) 31) (ASH (LOGAND 4026531840
 (R 4520 8 ST)) -1) (ASH (MOD (ASH (R 4520 8 ST) -27) 2) 26) (ASH (MOD (ASH (R 4520 8 ST) -28) 2) 25) (ASH (LOGAND 251658240 (R 4520
 8 ST)) -3) (ASH (MOD (ASH (R 4520 8 ST) -23) 2) 20) (ASH (MOD (ASH (R 4520 8 ST) -24) 2) 19) (ASH (LOGAND 15728640 (R 4520 8 ST))
 -5) (ASH (MOD (ASH (R 4520 8 ST) -19) 2) 14) (ASH (MOD (ASH (R 4520 8 ST) -20) 2) 13) (ASH (LOGAND 983040 (R 4520 8 ST)) -7) (ASH
 (MOD (ASH (R 4520 8 ST) -15) 2) 8)) -8) 24) (ASH (LOGIOR (ASH (MOD (ASH (R 4520 8 ST) -16) 2) 31) (ASH (LOGAND 61440 (R 4520 8 ST))
 15) (ASH (MOD (ASH (R 4520 8 ST) -11) 2) 26) (ASH (MOD (ASH (R 4520 8 ST) -12) 2) 25) (ASH (LOGAND 3840 (R 4520 8 ST)) 13) (ASH (MOD
 (ASH (R 4520 8 ST) -7) 2) 20) (ASH (MOD (ASH (R 4520 8 ST) -8) 2) 19) (ASH (LOGAND 240 (R 4520 8 ST)) 11) (ASH (MOD (ASH (R 4520 8
 ST) -3) 2) 14) (ASH (MOD (ASH (R 4520 8 ST) -4) 2) 13) (ASH (MOD (R 4520 8 ST) 16) 9) (ASH (ASH (R 4520 8 ST) -31) 8)) -8)) (R (+
 4376 (* 8 (R 4536 8 ST)) (* 8 (- (R 4528 8 ST)))) 8 ST)) -40) 256) -2))
 (ASH (MOD (ASH (MOD (ASH (LOGXOR (LOGIOR (ASH (ASH (LOGIOR (ASH (MOD (ASH (R 4520 8 ST) 0) 2) 31) (ASH (LOGAND 4026531840 (R 4520
 8 ST)) -1) (ASH (MOD (ASH (R 4520 8 ST) -27) 2) 26) (ASH (MOD (ASH (R 4520 8 ST) -28) 2) 25) (ASH (LOGAND 251658240 (R 4520 8 ST))
 -3) (ASH (MOD (ASH (R 4520 8 ST) -23) 2) 20) (ASH (MOD (ASH (R 4520 8 ST) -24) 2) 19) (ASH (LOGAND 15728640 (R 4520 8 ST)) -5) (ASH
 (MOD (ASH (R 4520 8 ST) -19) 2) 14) (ASH (MOD (ASH (R 4520 8 ST) -20) 2) 13) (ASH (LOGAND 983040 (R 4520 8 ST)) -7) (ASH (MOD (ASH
 (R 4520 8 ST) -15) 2) 8)) -8) 24) (ASH (LOGIOR (ASH (MOD (ASH (R 4520 8 ST) -16) 2) 31) (ASH (LOGAND 61440 (R 4520 8 ST)) 15) (ASH
 (MOD (ASH (R 4520 8 ST) -11) 2) 26) (ASH (MOD (ASH (R 4520 8 ST) -12) 2) 25) (ASH (LOGAND 3840 (R 4520 8 ST)) 13) (ASH (MOD (ASH (R
 4520 8 ST) -7) 2) 20) (ASH (MOD (ASH (R 4520 8 ST) -8) 2) 19) (ASH (LOGAND 240 (R 4520 8 ST)) 11) (ASH (MOD (ASH (R 4520 8 ST) -3)
 2) 14) (ASH (MOD (ASH (R 4520 8 ST) -4) 2) 13) (ASH (MOD (R 4520 8 ST) 16) 9) (ASH (ASH (R 4520 8 ST) -31) 8)) -8)) (R (+ 4376 (* 8
 (R 4536 8 ST)) (* 8 (- (R 4528 8 ST)))) 8 ST)) -40) 256) -2) 32) -1)
 (ASH (MOD (ASH (MOD (ASH (LOGXOR (LOGIOR (ASH (ASH (LOGIOR (ASH (MOD (ASH (R 4520 8 ST) 0) 2) 31) (ASH (LOGAND 4026531840 (R 4520
 8 ST)) -1) (ASH (MOD (ASH (R 4520 8 ST) -27) 2) 26) (ASH (MOD (ASH (R 4520 8 ST) -28) 2) 25) (ASH (LOGAND 251658240 (R 4520 8 ST))
 -3) (ASH (MOD (ASH (R 4520 8 ST) -23) 2) 20) (ASH (MOD (ASH (R 4520 8 ST) -24) 2) 19) (ASH (LOGAND 15728640 (R 4520 8 ST)) -5) (ASH
 (MOD (ASH (R 4520 8 ST) -19) 2) 14) (ASH (MOD (ASH (R 4520 8 ST) -20) 2) 13) (ASH (LOGAND 983040 (R 4520 8 ST)) -7) (ASH (MOD (ASH
 (R 4520 8 ST) -15) 2) 8)) -8) 24) (ASH (LOGIOR (ASH (MOD (ASH (R 4520 8 ST) -16) 2) 31) (ASH (LOGAND 61440 (R 4520 8 ST)) 15) (ASH
 (MOD (ASH (R 4520 8 ST) -11) 2) 26) (ASH (MOD (ASH (R 4520 8 ST) -12) 2) 25) (ASH (LOGAND 3840 (R 4520 8 ST)) 13) (ASH (MOD (ASH (R
 4520 8 ST) -7) 2) 20) (ASH (MOD (ASH (R 4520 8 ST) -8) 2) 19) (ASH (LOGAND 240 (R 4520 8 ST)) 11) (ASH (MOD (ASH (R 4520 8 ST) -3)
 2) 14) (ASH (MOD (ASH (R 4520 8 ST) -4) 2) 13) (ASH (MOD (R 4520 8 ST) 16) 9) (ASH (ASH (R 4520 8 ST) -31) 8)) -8)) (R (+ 4376 (* 8
 (R 4536 8 ST)) (* 8 (- (R 4528 8 ST)))) 8 ST)) -40) 256) -2) 2) 4))
```

While the first argument of the LOGIOR is easy to bound the second and third are problematic. Ainni bounds the LOGIOR to $[0, 63]$ in less than one hundredth of a second on a MacBook Pro laptop with a 2.6 GHz Intel Core i7 processor.

By the way, the second argument of the LOGIOR above actually lies in $[0, 15]$ and the third in $[0, 16]$. But proving those two bounds with, say, `arithmetic-5/top`, takes about 33 seconds each, without Ainni and meta-<. But the main point is that Ainni *infers* a correct bound.

⁶We anticipate that any ISA employing Stateman's byte-addressed memory would use twos-complement arithmetic.

6 Syntactic Simplification of MOD Expressions

Machine arithmetic introduces many MOD expressions in which the second argument is constant. Stateman provides the extended metafunction `meta-MOD` that implements the following rules, where i , j , and k are natural constants. The function also uses a concept called “syntactic integer” realized by a function which takes the quotation of a term and determines whether it is obviously integer valued. For example, a sum expression is a syntactic integer provided the two arguments are syntactic integers, an ASH expression is a syntactic integer provided the first argument is, and a LOGAND expression is a syntactic integer regardless of the shape of the arguments. In the rules below, x, x_1, \dots, x_j must be syntactic integer expressions.

- $(\text{MOD } x \ 0) = x$
- $(\text{MOD } i \ k)$ can be computed if both arguments are constants
- $(\text{MOD } (\text{MOD } z \ j) \ k) = (\text{MOD } z \ j)$, if $j \leq k$
- $(\text{MOD } (\text{MOD } x \ j) \ k) = (\text{MOD } x \ k)$, if k divides j
- $(\text{MOD } (\text{R } a \ i \ st) \ k) = (\text{R } a \ i \ st)$, if $256^i \leq k$
- $(\text{MOD } (+ \ x_1 \ \dots \ (\text{MOD } x \ j) \ \dots \ x_j) \ k) = (\text{MOD } (+ \ x_1 \ \dots \ x \ \dots \ x_j) \ k)$, if k divides j
- $(\text{MOD } x \ k) = x$, if Ainni claims the upper bound of x is below k

The last rule is not only applied to the argument of `meta-MOD` but also to the output of the second-to-last rule.

Some of these rules are built into `arithmetic-5/top` but in the interests of speed, Stateman does not export `arithmetic-5/top` and does much arithmetic simplification in its metafunctions.

7 Some Details of Meta-R and Meta-!R

The most complicated of the metafunctions are `meta-R` and `meta-!R`, which use all of the functionality described above. The former is actually more complicated than the latter because the former deals with mixed read-over-write. We briefly discuss some design issues for these two functions, starting with the simpler, `meta-!R`, but we urge the interested reader to inspect the code in the Stateman book.

Since a successful application of `meta-!R` will transform $(!R \ a \ 'n \ v \ (\text{HIDE } st'))$ into $(\text{HIDE } (!R \ a \ 'n \ v \ st'))$, we must be careful not to fire the metafunction too soon: none of the subterms will be rewritten again! Thus `meta-!R` checks whether a or v contain terms that might still be rewritten, e.g., embedded IFs, unexpanded LAMBDA applications, or read-over-writes that have not yet been resolved. If such subterms are found, the metafunction does not fire and $(!R \ a \ 'n \ v \ (\text{HIDE } st'))$ continues to be subject to rewriting.

If we decide to fire, we remove all HIDEs in a and v ; remember they are probably arithmetic/logical expressions formed by the semantics of an instruction operating on data extracted from memory and thus (controversially) hidden. When we remove HIDEs we actually compute the depth of the deepest HIDE first and then copy only that far into the term so as to avoid re-copying a honsed term.

Then we dive through st' looking for a perfect shadow of a write to a of n bytes. This is actually a little more complicated than just looking for a deeper $(!R \ a \ n \ \dots)$ because the addresses may not be fully normalized. By using Ainni we can identify some non-identical addresses that are semantically equivalent in the current context. As we dive through st' looking for a shadowed assignment we also compute its depth, so we can come back and delete it without further interval analysis.

Moving on to `meta-R`, the main complication is mixed read-over-write. The question is, given `(R a 'n (!R b 'k v st))`, does part of the answer lie within `v` or not? `Ainni` can be used to handle many general overlap questions but we prefer not to use `Ainni` if simpler techniques apply. For example, if both `a` and `b` are constants we can just skip over this `!R` or extract the appropriate bytes from `v` (remember `n` and `k` are constants). But more generally, we ask whether `a` and `b` are offsets from some common address, e.g., `a` might be `(+ 8 sp)` and `b` might be `(+ 16 sp)` where `sp` is some expression denoting, say, the stack pointer. While neither address is constant we can still determine whether reading `n` bytes from `a` takes us into the region written, by doing arithmetic on the two constant offsets (8 and 16 in this example) and the constants `n` and `k`. When no common reference address can be found, we use `Ainni`. Space does not permit further description of mixed read-over-write and we urge the reader to see the `Stateman` code.

Furthermore, space does not permit discussion of the proof issues. But correctness, guards, and well-formedness guarantees are all proved. Probably the most interesting and difficult proofs concerned mixed read-over-write and the validity of removing a deeply buried perfectly shadowed write *without* being able to determine whether intervening writes also shadow it, i.e., how do you justify transforming

```
(!R a n v1
  (!R b k w
    (!R a n v2 st)))
```

to

```
(!R a n v1
  (!R b k w st))
```

without knowing the relations between `a`, `n`, `b` and `k`? The formalization of the general result we need is an inductively proved `LOCAL` lemma, named `LEMMA3` in `stateman22.lisp`, establishing the correctness of a function that deletes a perfectly shadowed write at an arbitrary depth. `LEMMA3` is used in the proof of `META-!R-CORRECT`.

8 Memoization

We have experimented with memoization of the metafunctions introduced by `Stateman`. Memoization is theoretically useful in code proofs because the same symbolic state might be produced on different paths through the code. In addition, the contents of the same addresses might be read multiple times from the same state. On the other hand, memoization imposes an overhead and is thus not always worthwhile.

Memoization hits most often if all of the arguments are *honsed* rather than *consed*. For example, if `f` is memoized and one has typed `(f '(a . b))` at the top-level, then the value of `f` on that cons pair is stored in the hash table for `f`. But if one then types `(f (cons 'a 'b))` the memoized answer is not found and `f` is recomputed. In Common Lisp terms, the argument must be `EQ` not `EQUAL`. All of the state expressions produced by our metafunctions are *honsed* and thus uniquely represented. But this alone will not make `(memoize 'meta-R)`, for example, particularly useful.

First, memoization cannot be applied to an extended metafunction because one of the arguments is the `ACL2` (live) state. So `meta-R`, which takes `state` as an argument (because it is a requirement of extended metafunctions) but which ignores `state`, is defined in terms of a wrapper, `memoizable-meta-R` which does not take `state` and which takes only the type-alist from the metafunction context, not the whole context.

Second, the term argument of `meta-R` is of the form `(R a 'n (HIDE st'))` and typically came from simplifying some R-term in the model. The `(HIDE st')` is honsed because it was produced by one of our metafunctions. But the rest of the term is not. So we `hons-copy` it before calling the wrapper. These `hons-cops` are not as expensive as they may seem because the (very large) states and values extracted from them are already honsed.

Third, we must similarly `hons-copy` the type-alist.

Thus,

```
(defun meta-R (x mfc state)
  (declare (xargs :stobjs (state)
                  :guard (pseudo-term p x))
            (ignore state))
  (memoizable-meta-R (hons-copy x)
                     (hons-copy (mfc-type-alist mfc))))
```

Experiments have indicated that it is not worthwhile memoizing `meta-I`, `meta-S`, `meta-!I` or `meta-!S`: they are too simple. We have settled on:

```
(memoize 'memoizable-meta-r)
(memoize 'memoizable-meta-!r)
(memoize 'memoizable-meta-mod)
(memoize 'memoizable-meta-<)
```

While Ainni is an obvious candidate for memoization, the functions above include all of Ainni's callers so it is not worthwhile.

Finally, when a metafunction fires – even a metafunction with a well-formedness guarantee – the output is put into *quote normal form* by which we mean all ACL2 primitives applied to constants are evaluated to constants. That is, `(CONS '1 '2)` is not in quote normal form, but `'(1 . 2)` is. This reduction to quote normal form is done by applying the empty substitution to the term with the ACL2 utility `sublis-var1`. We have found it worthwhile to memoize this function, but only when the substitution is empty and the form being normalized is hidden (and thus probably one produced by our metafunctions and thus honsed).

```
(memoize 'sublis-var1
        :condition '(and (null alist)
                          (consp form)
                          (eq (car form) 'HIDE)))
```

(Future Work) More experimentation must be done before we are comfortable with these decisions. In addition, it might be practical to make well-formedness guarantees ensure quote normal form.

9 Preliminary Performance Results

We have tested Stateman on only one very stressful example. Roughly put the setup for this example (which is not provided here) is as follows: Using the state provided by Stateman, we defined an ISA for a register machine that provides conventional but realistic arithmetic/logical functionality, addressing

modes, and control flow. We then implemented a compiler from a subset of ACL2 into this ISA. After allocating declared arrays, constants, etc., the compiler uses the rest of the memory to provide a call stack whose stack and frame pointers are among the earlier addresses. The compiler then compiles a system of ACL2 functions and a main program as though it were running on a stack machine, e.g., $(\text{LOGAND } x \ y)$ is compiled by compiling x and y so as to leave two items on the stack, and then laying down a block of code to pop those two items into temporary registers, apply the LOGAND instruction to those registers, and push the result. Addressing modes are used whenever possible to minimize the number of instructions needed. We then compiled an ACL2 implementation of the DES algorithm.⁷ The result is a code block of 15,361 instructions. We then ran an experimental version of Codewalker on this code.

Using Codewalker and the state management techniques described here, ACL2 explores the code above and generates both clock and semantic functions for DES.⁸

The largest symbolic state in the decompilation of the DES algorithm represents one path through the 5,280 instructions in the decryption loop. The state contains 2,158,895 function calls consisting of one call of !I and !S each and 58 calls of !R to distinct locations. That state expression also contains 459,848 calls of R and 1,698,987 calls of arithmetic/logical functions such as +, and *, LOGAND, LOGIOR, LOGXOR, ASH, and MOD. The values written are often very large. The largest value expression written is given by a term involving 147,233 function applications, 31,361 of which are calls of R and the rest are calls of arithmetic/logical functions.

We would like to be able to compare the performance of the current version of Stateman to older techniques (in which rewrite rules alone are used to canonicalize symbolic states) but Codewalker is unable to complete the exploration of our implementation of DES using those older techniques. The time it takes to symbolically execute successive instructions increases alarmingly, sometimes apparently exponentially (depending on the instruction being executed) as the state sizes increase. Of course, one might address that with better rewrite rules, metafunctions, etc., but that was the origin of the Stateman project.

However, we can provide some timing statistics on different versions of Stateman. The times shown are times taken to generate the clock and semantics functions of our DES implementation on a MacBook Pro laptop with a 2.6 GHz Intel Core i7 processor with 16 GB of 1600 MHZ DDR3 memory. Times are as measured by `time$` and reported as “realtime” on a otherwise unloaded machine.

Roughly put, guard verification saved 33 seconds, well-formedness guarantees saved 337 more seconds, honsing as opposed to consing the metafunction answers saved 124 more seconds even though no memoization was employed, and memoizing then saved 119 more seconds. Of particular interest is that well-formedness guarantees were an order of magnitude more effective than guard verification and that

⁷Warren Hunt provided the definitions of the ISA and the DES algorithm in ACL2.

⁸As of this writing the Codewalker exploration of DES does not perform its standard “projection” (the transformation of functions that describe the entire state to functions that describe the contents of specific state components) because ACL2 gets a stack overflow trying to handle states of such large size. **(Future Work)** Clearly, additional work is necessary on Codewalker and/or ACL2 itself to handle the terms being produced by Stateman.

honsing even without memoization was a win (presumably because less time was spent in allocation).

without guard verification, well-formedness guarantees, honsing or memoization	988 seconds
with guard verification but without well-formedness guarantees, honsing, or memoization	955 seconds
with guard verification and well-formedness guarantees, but without honsing or memoization	618 seconds
with guard verification, well-formedness guarantees, and honsing, but without memoization	494 seconds
with guard verification, well-formedness guarantees, honsing, and the memoization described	375 seconds

10 Acknowledgments

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References

- [1] S. Goel, W.A. Hunt & M. Kaufmann (2014): *Simulation and Formal Verification of x86 Machine-Code Programs that make System Calls*. In K. Claessen & V. Kuncak, editors: FM-CAD'14: Proceedings of the 14th Conference on Formal Methods in Computer-Aided Design, <http://www.cs.utexas.edu/users/hunt/FMCAD/FMCAD14/proceedings/final.pdf>, EPFL, Switzerland, pp. 91–98, doi:10.1109/FMCAD.2014.6987600.
- [2] M. Kaufmann, P. Manolios & J S. Moore, editors (2000): *Computer-Aided Reasoning: ACL2 Case Studies*. Kluwer Academic Press, Boston, MA.
- [3] M. Kaufmann, P. Manolios & J S. Moore (2000): *Computer-Aided Reasoning: An Approach*. Kluwer Academic Press, Boston, MA., doi:10.1007/978-1-4615-4449-4.
- [4] M. Kaufmann & J S. Moore (2014): *The ACL2 Home Page*. In: <http://www.cs.utexas.edu/users/moore/ac12/>, Dept. of Computer Sciences, University of Texas at Austin.
- [5] M. Kaufmann & J S. Moore (2015): *Well-Formedness Guarantees for ACL2 Metafunctions and Clause Processors*. In: (submitted for publication).
- [6] H. Liu & J S. Moore (2004): *Java Program Verification via a JVM Deep Embedding in ACL2*. In K. Slind, A. Bunker & G. Gopalakrishnan, editors: *17th International Conference on Theorem Proving in Higher*

- Order Logics: TPHOLs 2004, Lecture Notes in Computer Science 3223*, Springer, pp. 184–200, doi:10.1007/978-3-540-30142-4_14.
- [7] J S. Moore & M. Martinez (2009): *A Mechanically Checked Proof of the Correctness of the Boyer-Moore Fast String Searching Algorithm*. In: *Engineering Methods and Tools for Software Safety and Security (Proceedings of the Martoberdorf Summer School, 2008)*, IOS Press, pp. 267–284, doi:10.3233/978-1-58603-976-9-267.
 - [8] Magnus O. Myreen (2009): *Formal verification of machine-code programs*. Ph.D. thesis, University of Cambridge.
 - [9] Magnus O. Myreen, Konrad Slind & Michael J. C. Gordon (2012): *Decompilation into Logic Improved*. In: *Formal Methods in Computer-Aided Design (FMCAD), 2012*, pp. 78–81.
 - [10] E. Toibazarov (2013): *An ACL2 Proof of the Correctness of the Preprocessing for a Variant of the Boyer-Moore Fast String Searching Algorithm*. Honors Thesis, Computer Science Dept., University of Texas at Austin. See <http://www.cs.utexas.edu/users/moore/publications/toibazarov-thesis.pdf>.